Kolmogorov-Sinai entropy and dissipation in classical Hamiltonian systems

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Many connections between physics and information theory have been revealed since the development of classical information theory by Shannon. A key concept in this connection is entropy, which represents the amount of information transferred to the observer who performs measurements in an experiment. Statistical mechanics is a physical theory deeply connected to information by Jaynes Maximum Entropy principle, which defines equilibrium probability distributions as the ones that maximizes entropy under some physical constraints. In this way, these distributions are the less unbiased probabilities that can be assignment to an event. Following this path, the dissipated energy in a classical Hamiltonian process (also known as the thermodynamic entropy production) was connected to the relative entropy between the forward and backward probability densities. A recent work by Still et al. has revealed that energetic inefficiency and model inefficiency are equivalent concepts in Markovian processes, where the latter is defined as the difference in mutual information that the systems state shares with the future and past environmental variables. This raises the question whether model unpredictability and energetic inefficiency are connected in the framework of classical physics. The aim of this study is to connect the concepts of random behavior of a classical Hamiltonian system with its energetic inefficiency. The random behavior of a classical system is quantified by the Kolmogorov-Sinai entropy associated with its dynamics, an information-theoretic approach to chaos, whereas energetic inefficiency is measured by the dissipated work.