$q$-statistics and $\kappa$-statistics: a comparative study

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Two deformed structures (among others) are used in generalized statistics, namely, the Tsallis [C. Tsallis, J. Stat. Mech. 52, 479 (1988)] and the Kaniadakis [G. Kaniadakis, Physica A 296, 405 (2001)] formulations. Both of them are based on generalizations of the exponential function: the $q$-exponential ($\exp_q(x) \equiv [1+(1-q)x]^{1/(1-q)}$) for the former, and the $\kappa$-exponential ($\exp_\kappa(x) \equiv [\sqrt{1+\kappa^2x^2 + \kappa x}]^{1/\kappa}$) for the later, and their corresponding inverse functions, the generalized logarithms. Both generalized exponentials asymptotically behave as power laws. These functions yield generalized hyperbolic and trigonometric functions, generalized algebraic operations and generalized calculus. Both cyclic functions, for instance, have a finite number of roots, but for different reasons. The $\kappa$-sine and $\kappa$-cosine functions are defined in a limited range of the real domain. The $q$-sine and $q$-cosine are defined in $\mathbb{R}$, but with a limited number of oscillations, and asymptotically goes to zero (for $q > 1$). Besides, the amplitude of the $\kappa$-cyclic functions are constant ($[-1,1]$), while the amplitude of the $q$-cyclic functions varies (a decay for $q > 1$, an increase of amplitude for $q < 1$). In general, the formalisms recover the usual one within a proper limit $q \to 1$ and $\kappa \to 0$. The aim of this preliminary study is to compare the main properties of these two formulations, and understand their differences, similarities, and limitations.

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