Phase space quantization procedures and the Wigner function formulation

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The concept of phase space arises naturally from the Hamiltonian formulation of classical mechanics and plays an important role in the transition from classical physics to quantum theory. In this context, one of the interesting approaches to the quantization problem is that which formulates itself on the phase space of the physical system under consideration. Several authors (van Hove, Souriau, Streater, Berezin, Prigogine, Prugovecki, Segal) have suggested a quantization mapping that we write as:

\[ \hat{f} = i X_f + f - p.k \frac{\partial f}{\partial p} - q.k' \frac{\partial f}{\partial q} \quad (\hbar = 1) \]  

(1)

where \( \hat{f} \) is an operator and \( X_f \) is a tangent vector field on the phase space associated with the function \( f \) for some vector field \( X \), and \( k \) and \( k' \) are constants to be determined by mathematical and physical considerations. On the other hand, Weyl's law involves the star product and it gives an unitary operator as:

\[ a_w^* = a_w(q, p) \exp \left[ \frac{i}{2} (\partial_q \partial_p - \partial_p \partial_q) \right] ; \quad (\hbar = 1) \]

(2)

where the arrows over the vector fields \( \partial_q, \partial_p \) denote that a given vector field acts only the function standing on the left or on the right side of the vector field. Studies of the representations of the Galilei group in a manifold with phase space content have been developed since long ago. In this communication, we explore the quantization schemes (1) e (2) in connection with the Lie algebra of the Galilei group in order to determine a possible relation between these procedures; the generators of the Galilei group and its Casimir Invariants are analyzed and, using schemes (1) and (2), we show a procedure to obtain the quasi-distribution Wigner function formalism.