Greens function construction for quantum graphs based on their adjacency matrices

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A graph consists of a set of vertices connected by edges. By adding a metric and attaching leads we turn it into a scattering system. A ubiquitous technique to analyze scattering systems is the Green’s functions. In a previous work [1], it was shown that the exact Green’s function for an arbitrarily large (although finite) quantum graph is given as a sum over scattering paths, where local quantum effects are taking into account through the reflection and transmission scattering amplitudes. To deal with general graphs, two simplifying procedures were developed, i.e., regrouping of paths into families of paths and the separation of a large graph into subgraphs. However, for less symmetrical graphs with complicated topologies as, for instance, random graphs, it can become cumbersome to choose the subgraphs and the families of paths. Moreover, the choice of families of paths is not unique. In this work, a more general procedure to construct the energy domain Green’s function for a quantum graph based on its adjacency matrix is developed. We also discuss that this technique can be used to study scattering quantum walks. The present construction has the advantage that it can be applied directly for any graph, going from regular to random topologies, and provides a unique way to determine the set of families of paths for a given graph.