Scaling law in bifurcation points in the Logistic Map

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The Logistic Map $x_{n+1} = rx_n(1-x_n)$, $x_n \in [0, 1]$ and $r \in [0, 4]$ is one of the simplest and mostly widely known dynamical system. It shows a variety of behaviour present in more sophisticated systems. From its critical point, one can generate a set of iterative functions called Supertracks Functions $s_n(r)$. These functions are bounds states of the dynamics of the bifurcation diagram of the logistic map. We use this set of functions and its derivatives to generate two sets of sequences: the values of the orbit points and its tendency (derivative) of slope. By comparing and analyzing these sequences and their own elements, one can obtain the rules of these sequences without the need of the iteration process, and the convergence of them allows the understanding of the dynamics of the orbits. The firsts points $(r, x_n \rightarrow \infty)$ of bifurcation in the bifurcation diagram of the logistic map are well known, but numerically, by iteration process, are not reachable: the more one iterates the orbits, the closer the orbits become from the bifurcation point, without arriving there. From one point of view this sounds strange, because the Lyapunov Exponent tends to zero in the bifurcation points, but the orbits do not reach these points. From another point of view, these points can be never there, because they do not move. The scaling law allows a better understanding of this behaviour.